Single-Particle Interference
Can Witness Bipartite Entanglement

Torsten Scholak$^1$ $^3$  Florian Mintert$^2$ $^3$  Cord A. Müller$^1$

March 13, 2008
Motivation

- double slit experiment

\[ \begin{array}{c}
\text{source} \\
A \\
B \\
\text{detector}
\end{array} \]

\text{perfect visibility} \quad \text{perfect knowledge}^*

- \textit{atoms} provide information about \textit{photon path}

\textit{photon} gathers information about \textit{atoms}

What Information Can Be Gained by Scattering a Probe?

- correlations?
- distinguish between *classical* and *quantum* correlations?

\[ \sim \text{witness entanglement} \]
Entanglement Witnesses

- an observable $W$ is an **entanglement witness**:\[\Leftrightarrow\]

\[
\langle W \rangle = \text{tr}\{\rho_t W\} \begin{cases} 
\geq 0, & \text{for all separable states } \rho_t \\
< 0, & \text{for at least one entangled state}
\end{cases}
\]

\[\Rightarrow \text{negative outcome: } \rho_t \text{ entangled!}\]

\[\sim \text{look for characteristic sign changes!}\]

\[\langle W \rangle = 0\]

Probe-Target Interaction

\[ \rho = \rho_p \otimes \rho_t \]
Probe-Target Interaction

\[ \rho = \rho_p \otimes \rho_t \]

\[ T = e^{i\phi_A} T_A + e^{i\phi_B} T_B \]

\[ \phi = \phi_A - \phi_B \]
Probe-Target Interaction

\[ \rho = \rho_p \otimes \rho_t \]

\[ T = e^{i\phi_A} T_A + e^{i\phi_B} T_B \]

\[ \phi = \phi_A - \phi_B \]

\[ I_0 \geq 0, \text{ no witness} \]

\[ |\delta I(\phi)| \leq l_0, \text{ witness?} \]
Central Result: Connection of Pattern and Witness

⇒ designated entanglement witness:

reading off interference contribution at $\phi = 0$

$$\delta I(0) = \langle T_B^\dagger P T_A + T_A^\dagger P T_B \rangle$$
Central Result: Connection of Pattern and **Witness**

⇒ **designated entanglement witness:**

\[ \delta I(0) = \text{tr}_{p \& t} \left\{ (\rho_p \otimes \rho_t) \left( T_B^\dagger P T_A + T_A^\dagger P T_B \right) \right\} \]
Central Result: Connection of Pattern and **Witness**

⇒ designated entanglement witness:

reading off **interference** contribution
at $\phi = 0$

\[
\delta I(0) = \text{tr}_t \left\{ \rho_t \text{ tr}_p \left[ \rho_p \left( T_B^\dagger P T_A + T_A^\dagger P T_B \right) \right] \right\}
\]
Central Result: Connection of Pattern and **Witness**

⇒ **designated entanglement witness:**

- reading off interference contribution at $\phi = 0$
- measuring exp. value of observable on the target system

\[
\delta I(0) = \text{tr}_t \left\{ \rho_t \text{tr}_p \left[ \rho_p \left( T_B^\dagger P T_A + T_A^\dagger P T_B \right) \right] \right\} = \text{tr}_t \left\{ \rho_t M \right\}
\]
Central Result: Connection of Pattern and \textbf{Witness}

⇒ designated entanglement witness:

reading off \textbf{interference} contribution at $\phi = 0$ \iff measuring exp. value of \textbf{observable} on the target system

$$\delta I(0) = \text{tr}_t \left\{ \text{tr}_p \left[ \rho_p \left( T_B^\dagger P T_A + T_A^\dagger P T_B \right) \right] \right\} = \text{tr}_t \left\{ \rho_t M \right\}$$

⇒ criterion:

\begin{align*}
\delta I(0) \begin{cases} 
> 0 : & \text{no detection} \\
< 0 : & \rho_t \text{ entangled!}
\end{cases}
\end{align*}
Quantum Optics Realization of $W_-$

- single probe photon
- elastic single scattering
- 2 atoms carry target qubit pair
- unpolarized incident beam, no polarizer ($P = 1$)

- for detection at right angles to incidence ($\hat{k} \perp \hat{k}'$):
  \[
  M = 2 \left( 1 - 2 |\psi_-\rangle \langle \psi_-| \right)
  \]
  witnesses the singlet state $|\psi_-\rangle = \frac{1}{\sqrt{2}} \left( |= =, = = \rangle - |= =, = = \rangle \right)$

- works for all entangled states $|\psi\rangle$ with $|\langle \psi | \psi_- \rangle|^2 > \frac{1}{2}$
Discussion

- single-particle interference is sensitive for entanglement
- mapping of a two-particle observable to a single-particle observable
- proof-of-concept model found in quantum optics
- arXiv:0710.0825:
Thank You for Your Attention!