Spectral Signatures of Excitation Transport in Ultra-Cold Rydberg Gases

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What effect does DIPOLE BLOCKADE have on Rydberg gas spectral statistics?

- Rydberg atoms are randomly distributed and interact via strong long-range forces $V \sim R^{-3}$

⇒ dynamics are fundamentally different from short-range interaction in ordered ensembles

- dipole blockade effect can create ordered structures from unordered ensembles

⇒ dipole blockade should affect spectral, eigenstate, and transport statistics strongly

Model

- spherical frozen gas cloud with N Rydberg atoms
- typical (unblockaded) nearest neighbor distance:

$$(2\pi\rho)^{-1/3} \simeq 10\mu\text{m}$$

- one $n\text{P}_{\frac{3}{2},m_j=|\frac{3}{2}|}$ among $(N-1)$ $n\text{S}_{\frac{1}{2},m_j=|\frac{1}{2}|}$
- single-excitation basis states: $|\text{SS}\ldots\text{P}\ldots\text{S}\rangle = |i\rangle$
- Hamiltonian matrix elements:

$$H_{ij} = \langle i|H|j\rangle = (1-\delta_{ij})\,\beta\,\tfrac{9\sqrt{3}}{8\pi}\left(3\cos^2\Theta_{ij}-1\right)R_{ij}^{-3},$$

β empirical constant

- choose units of length and energy such that $\rho \equiv \beta \equiv 1$
- Rydberg blockade radius: r_b
- atoms uniformly distributed barring exclusion sphere:

$$r_\text{b} \le R_{ij} \le d = 2\sqrt[3]{3N/4\pi}$$

- $r_\text{b} = 0$: heavy-tailed H_{ij}, $f_{H_{ij}}(h) \sim_{|h|\to\infty} N^{-1}|h|^{-2}$
- $r_\text{b} > 0$: $f_{H_{ij}}$ truncated, $|H_{ij}| \le 9\sqrt{3}/4\pi r_\text{b}^3$

Pairs

- separated by much less than $(2\pi\rho)^{-1/3}$
- pair-localized state:

$$\tfrac{1}{\sqrt{2}}\left(|i\rangle \pm |j\rangle\right)$$

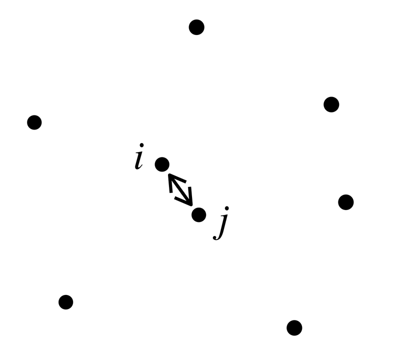
- eigenvalues:

$$\Lambda_\pm \simeq \pm H_{ij},$$

uncorrelated

- $r_\text{b} = 0$: heavy-tailed density of states,

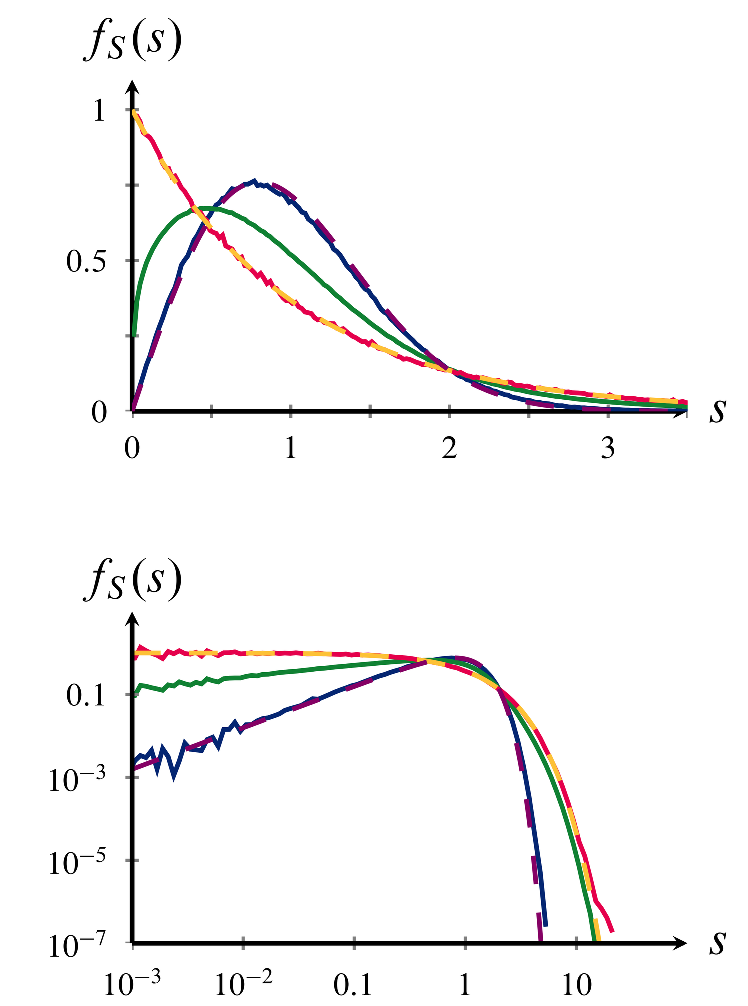
$$f_\Lambda(\lambda) \sim_{|\lambda|\to\infty} |\lambda|^{-2}$$

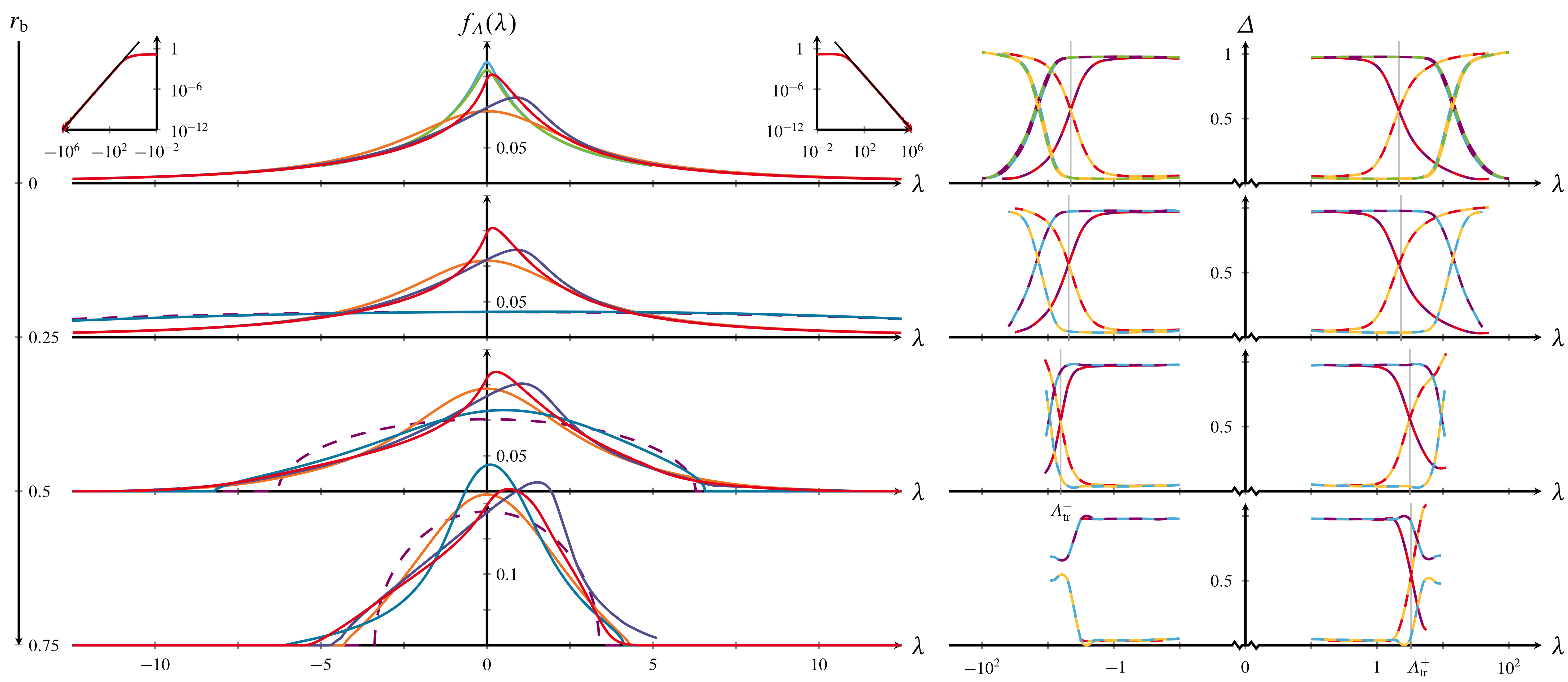


Level Spacings

- $f_S(s)\,\mathrm{d}s$: probability to sample two adjacent eigenvalues Λ_ν, $\Lambda_{\nu+1}$ separated by $S \in (s, s+\mathrm{d}s)$
- unfolded: $\overline{S} = \int_0^\infty s f_S(s)\,\mathrm{d}s = 1$
- plotted on the right for $N = 10^4$, $r_\text{b} = 0$
  - Poisson: uncorrelated, localized
  - Wigner-Dyson: GOE, extended
  - band wings ($|\Lambda| \ge 100$): $\sim$ Poisson
  - band center ($|\Lambda| \le 0.2$): $\sim$ GOE
  - intermediate ($0.2 < |\Lambda| < 100$): mixed with algebraic level repulsion for small s
- root-mean-square deviation:

$$\Delta[f_S, g_S] = \tfrac{1}{n}\left(\int_0^\infty [f_S(s)-g_S(s)]^2\,\mathrm{d}s\right)^{1/2}$$





- plotted for $N = 10^4$
- f_Λ is heavy-tailed and slightly skewed
- the stronger the blockade, the smaller the bandwidth
- critical energies $\Lambda_\text{tr}^\pm$ mark transition between Poisson and Wigner-Dyson statistics
- $\Lambda_\text{tr}^+ > |\Lambda_\text{tr}^-|$
- no transition for large r_b, statistics closer to Wigner-Dyson at all energies.
- model without Euclidean correlations: localized states are also suppressed, but at energies with higher absolute value

Random Matrix Ensembles

- Gaussian Orthogonal: $f_{M_{ij}}(m) = \frac{1}{\sqrt{\pi\sigma^2}}\mathrm{e}^{-m^2/\sigma^2}$ with variance $\frac{\sigma^2}{2} = \frac{27b^6\left[5+b^2(-9+4b)+6\ln b\right]}{160N^2\left[-2+b^2(9-8b+b^4)\right]}$ and $b = \frac{\sqrt[3]{6N/\pi}}{r_\text{b}}$
- 1-Stable: $f_{M_{ij}}(m) = \frac{1}{N}\left(m^2 + \frac{\pi^2}{N^2}\right)^{-1}$

Conclusions

- transitions from delocalized to localized states
- more blockade → more order → GOE statistics → less localization → more diffusion
- less blockade → less order → 1SE statistics → more localization → less diffusion
- Euclidean correlations cause localization and skewness at the band center
- low concentration approximation for r_b small, high concentration approximation for r_b large.

Selfconsistent Locator Expansion

- density of states:

$$f_\Lambda(\lambda) = \frac{1}{N}\overline{\operatorname{Tr}[\delta(\lambda - H)]} = -\frac{1}{\pi}\lim_{\varepsilon\to 0^+}\operatorname{Im}\overline{G_{00}(\lambda + \mathrm{i}\varepsilon)},$$

$$\overline{G_{00}(z)} = \overline{\langle 0|(z-H)^{-1}|0\rangle}$$

- locator expansion:

$$z\,\overline{G_{00}(z)} = 1 + \sum_{l=1}^{\infty}\sum_{k=l}^{\infty}\rho^l z^{-(k+1)}\int\cdots\int_{D_l}\sum_{i\in I_{kl}} H_{0i_1}H_{i_1i_2}\cdots H_{i_k0}\,\mathrm{d}\boldsymbol{R}_1\cdots\mathrm{d}\boldsymbol{R}_l$$

- low concentration approximation:
  - first order: includes no Euclidean correlations!

$$z\,\overline{G_{00}(z)} = 1 + \rho F_1\left(\overline{G_{00}(z)}\right)$$

  - second order: includes some Euclidean correlations!

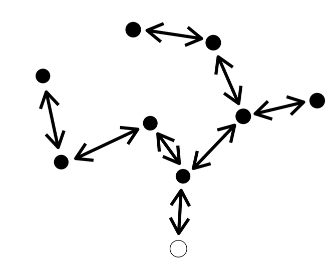
$$z\,\overline{G_{00}(z)} = 1 + \rho F_1\left(\overline{G_{00}(z)}\right) + \rho^2 F_2\left(\overline{G_{00}(z)}\right)$$

- high concentration approximation: includes different Euclidean correlations!

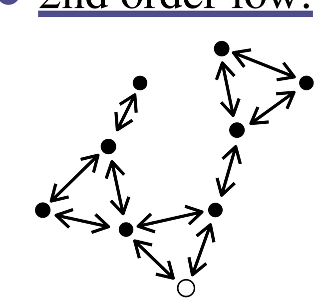
$$z\,\overline{G_{00}(z)} \simeq 1 + \sum_{l=1}^{\infty}\rho^l z^{-(l+1)}\int\cdots\int_{D_l} H_{01}z\,\overline{G_{11}(z)}\cdots H_{l0}z\,\overline{G_{00}(z)}\,\mathrm{d}\boldsymbol{R}_1\cdots\mathrm{d}\boldsymbol{R}_l$$

Diagrams

- 1st order low:



- 2nd order low:



- high: