# Single-Particle Interference Can Witness Bipartite Entanglement

Torsten Scholak[1 3] Florian Mintert[2 3] Cord A. Müller[1]

1 UNIVERSITÄT BAYREUTH

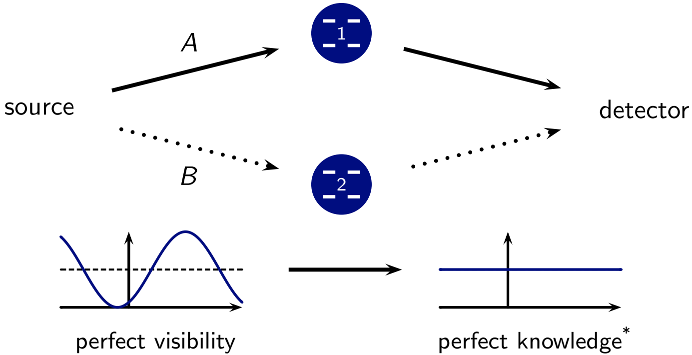
2 mpipks

3

March 13, 2008

# Motivation

- double slit experiment

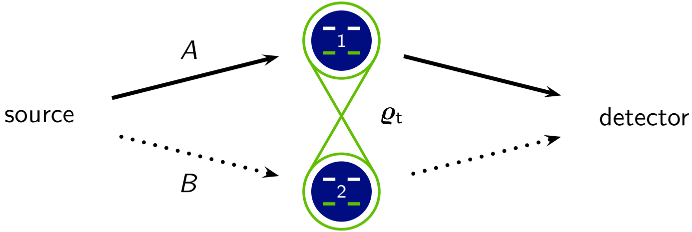



- *atoms* provide information about *photon path*

$\overset{?}{\rightsquigarrow}$ *photon* gathers information about *atoms*

cf. U. Eichmann, Phys. Rev. Lett. **70**, 2359 (1993)

# What Information Can Be Gained by Scattering a Probe?




- correlations?
- distinguish between *classical* and *quantum* correlations?

$\rightsquigarrow$ **witness entanglement**

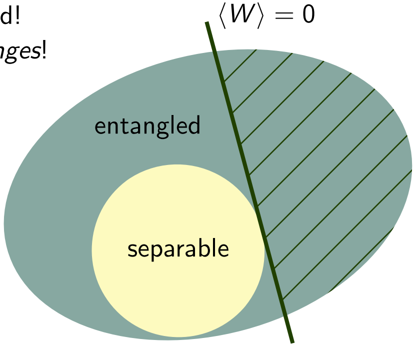
# Entanglement Witnesses

- an observable $W$ is an **entanglement witness**:$\Leftrightarrow$

$$\langle W \rangle = \mathrm{tr}\{\varrho_\mathrm{t} W\} \begin{cases} \geq 0, & \text{for all separable states } \varrho_\mathrm{t} \\ \mathbf{< 0}, & \text{for at least one entangled state} \end{cases}$$

$\Rightarrow$ **negative** outcome: $\varrho_\mathrm{t}$ entangled!

$\rightsquigarrow$ look for *characteristic sign changes*!



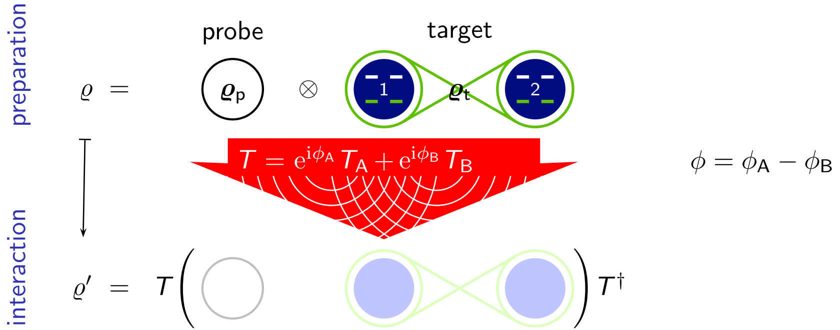

cf. P. Horodecki, Phys. Lett. A **232**, 33 (1997)

# Probe-Target Interaction

preparation

$\varrho \; = $ 

# Probe-Target Interaction

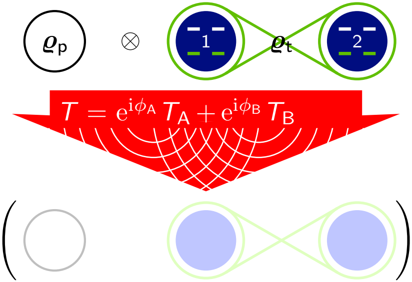


# Probe-Target Interaction

preparation

probe target

$$\varrho = \varrho_\mathrm{p} \otimes \varrho_\mathrm{t}$$




$$T = \mathrm{e}^{\mathrm{i}\phi_\mathsf{A}}\, T_\mathsf{A} + \mathrm{e}^{\mathrm{i}\phi_\mathsf{B}}\, T_\mathsf{B}$$

$$\phi = \phi_\mathsf{A} - \phi_\mathsf{B}$$

interaction

$$\varrho' = T\Big(\quad\Big)T^\dagger$$

projector $P$

probe

measurement

$$\langle P\rangle' = \underbrace{\langle T_\mathsf{A}^\dagger P T_\mathsf{A}\rangle + \langle T_\mathsf{B}^\dagger P T_\mathsf{B}\rangle}_{I_0 \geq 0,\ \text{no witness}} + \underbrace{\left[\mathrm{e}^{\mathrm{i}\phi}\langle T_\mathsf{B}^\dagger P T_\mathsf{A}\rangle + \mathrm{e}^{-\mathrm{i}\phi}\langle T_\mathsf{A}^\dagger P T_\mathsf{B}\rangle\right]}_{|\delta I(\phi)| \leq I_0,\ \text{witness?}}$$

# Central Result: Connection of Pattern and **Witness**

$\Rightarrow$ **designated entanglement witness:**

$$\delta I(0) = \underbrace{\langle T_B^\dagger P T_A + T_A^\dagger P T_B \rangle}_{\text{reading off}\ \textbf{interference}\ \text{contribution at}\ \phi = 0}$$

# Central Result: Connection of Pattern and **Witness**

$\Rightarrow$ **designated entanglement witness:**

reading off
**interference** contribution
at $\phi = 0$

$$\delta I(0) \;=\; \mathrm{tr}_{\mathrm{p\&t}}\Big\{(\varrho_{\mathrm{p}} \otimes \varrho_{\mathrm{t}})\Big(T_{\mathrm{B}}^{\dagger} P T_{\mathrm{A}} + T_{\mathrm{A}}^{\dagger} P T_{\mathrm{B}}\Big)\Big\}$$

# Central Result: Connection of Pattern and **Witness**

⇒ **designated entanglement witness:**

reading off
**interference** contribution
at $\phi = 0$

$$\delta I(0) = \mathrm{tr}_{\mathrm{t}}\left\{ \varrho_{\mathrm{t}} \ \mathrm{tr}_{\mathrm{p}}\left[ \varrho_{\mathrm{p}} \left( T_{\mathrm{B}}^{\dagger} P T_{\mathrm{A}} + T_{\mathrm{A}}^{\dagger} P T_{\mathrm{B}} \right) \right] \right\}$$

# Central Result: Connection of Pattern and **Witness**

⇒ **designated entanglement witness:**

reading off **interference** contribution at $\phi = 0$ ⇔ measuring exp. value of **observable** on the target system

$$\underbrace{\delta I(0)}_{\text{reading off interference contribution at } \phi=0} = \mathrm{tr}_\mathrm{t}\left\{\varrho_\mathrm{t} \boxed{\mathrm{tr}_\mathrm{p}\left[\varrho_\mathrm{p}\left(T_\mathrm{B}^\dagger P T_\mathrm{A} + T_\mathrm{A}^\dagger P T_\mathrm{B}\right)\right]}\right\} = \mathrm{tr}_\mathrm{t}\left\{\varrho_\mathrm{t} \boxed{M}\right\}$$

# Central Result: Connection of Pattern and **Witness**

$\Rightarrow$ **designated entanglement witness:**

reading off
**interference** contribution
at $\phi = 0$

$\Leftrightarrow$

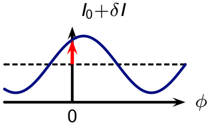
measuring
exp. value of **observable**
on the target system

$$\delta I(0) = \mathrm{tr_t}\left\{\varrho_\mathrm{t} \boxed{\mathrm{tr_p}\left[\varrho_\mathrm{p}\left(T_\mathrm{B}^\dagger P T_\mathrm{A} + T_\mathrm{A}^\dagger P T_\mathrm{B}\right)\right]}\right\} = \mathrm{tr_t}\left\{\varrho_\mathrm{t} \boxed{M}\right\}$$

$\Rightarrow$ **criterion:**

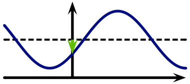
$\delta I(0)$

$> 0$ :



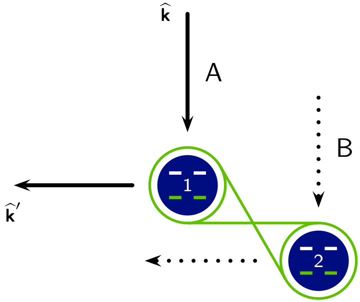

no detection

$< 0$ :



$\varrho_\mathrm{t}$ **entangled**!

# Quantum Optics Realization of $W_-$




- **single probe photon**
- elastic **single scattering**
- 2 atoms carry **target qubit pair**
- **unpolarized** incident beam,
  **no polarizer** $(P = \mathbb{1})$

- for detection at **right angles** to incidence $(\hat{\mathbf{k}} \perp \hat{\mathbf{k}}')$:

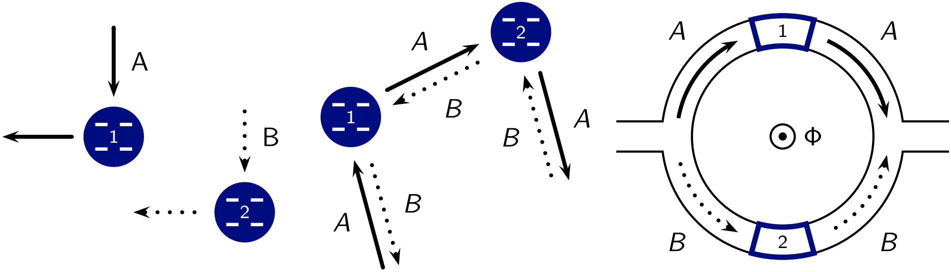
$$\boxed{M = 2\,(\mathbb{1} - 2|\Psi_-\rangle\langle\Psi_-|)}$$

witnesses the singlet state $|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\ ,\ \rangle - |\ ,\ \rangle\right)$

- works for all entangled states $|\Psi\rangle$ with $|\langle\Psi|\Psi_-\rangle|^2 > \frac{1}{2}$

# Discussion

- single-particle interference is sensitive for entanglement
- mapping of a two-particle observable to a single-particle observable
- proof-of-concept model found in quantum optics
- arXiv:0710.0825:



# Thank You for Your Attention!