A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

September 2008

EPL, **83** (2008) 60006
doi: 10.1209/0295-5075/83/60006

www.epljournal.org

Entanglement witnesses from single-particle interference

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received 22 April 2008; accepted in final form 29 July 2008
published online 10 September 2008

PACS 03.67.Mn – Entanglement measures, witnesses, and other characterizations
PACS 73.23.-b – Electronic transport in mesoscopic systems
PACS 42.25.Hz – Interference

Abstract – We describe a general method of realizing entanglement witnesses in terms of the interference pattern of a single quantum probe. After outlining the principle, we discuss specific realizations both with electrons in mesoscopic Aharonov-Bohm rings and with photons in standard Young's double-slit or coherent-backscattering interferometers.

How much information can be gained about separate parts of a composite quantum system by scattering a single probe? It has been known since the founding days of quantum mechanics [1] that there are correlations between the subcomponents of quantum systems that cannot be understood in terms of classical probabilities. More recently, the quest for theoretical tools that characterize entangled states [2–4] has been spurred by the prospect of utilizing the underlying quantum parallelism for a new era of information processing [5]. However, realizing these theoretical techniques in laboratory experiments is still a challenge [6–8], essentially because a multitude of observables has to be measured. Consequently, it is desirable to characterize entangled states with a minimal set of measurements.

In this letter, we study the case of a probe particle that gathers interferential information on the subsystems, which is then read out by a suitable measurement only on the probe. This principle has been used in the past, such as in Bragg scattering of X-rays by crystals, since certain types of correlations (such as positions on a lattice) can be inferred from the interference pattern of a quantum probe. It is by no means obvious, however, whether a single-particle interference pattern can distinguish subtle quantum correlations in entangled states from classical correlations in separable ones. It was shown in the context of mesoscopic solid-state systems that the visibility of conductance oscillations of double quantum dots can be sensitive to their entanglement [9]. Here we put this result in a broader context and show generally how the entanglement witness expectation value of bipartite qubit systems can be read off the two-way interference fringes of a single quantum probe. In essence, this method exploits quantum parallelism: if the probe particle is brought into a superposition of two states, each of which interacts with one of the subsystems, this single particle gains information about both subsystems and its interference pattern can reveal the desired information about entanglement.

Consider a bipartite quantum system with the two subsystems labeled 1 and 2. There are several techniques to determine whether their common state ϱ_{12} is entangled or not [10]. Bell inequalities, for example, are designed to distinguish quantum from classical correlations as predicted by local realism [2,11]. If the correlations between suitably chosen observables exceed a given threshold value, then the underlying quantum state is entangled [12,13]. However, a single inequality detects only rather few states, and there is even a multitude of entangled states that cannot be detected by any Bell test [14].

There are entanglement measures that detect every entangled state without state-dependent adjustment [3, 4,15]. However, they can only be evaluated if the entire density operator ϱ_{12} is known. Since quantum state tomography [16] requires a complete set of measurements, it becomes less and less feasible with increasing dimension of the subsystems' Hilbert spaces.

State tomography can be avoided with methods of direct measurement —at the expense of multiple simultaneous state preparation [17–22]. Here, some entanglement measures can be rephrased or approximated as the

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expectation value of collective observables on several identically prepared quantum states [8,23,24]. Both quantum state tomography and direct measurements are applicable for every quantum state, but this advantage of universality is gained by cutting back on experimental feasibility.

Entanglement witnesses [4], by contrast, require neither complete state tomography nor multiple parallel state preparation. By definition, a witness is an observable with positive expectation values $\langle W\rangle = \mathrm{tr}_{12}\{\varrho_{12}W\} \geqslant 0$ on all separable states, but negative expectation values on certain entangled states. A negative measurement outcome thus implies with certainty that the target state was entangled. A single witness (just like a single Bell inequality) cannot detect entanglement universally, but (in contrast to Bell inequalities) to any target state one can find a witness that detects its entanglement [4].

Still, a witness is a global observable on the combined target system, and it requires *several* local measurements on the subsystems to reconstruct the witness [25,26]. Our aim is now to evaluate the target witness expectation value $\langle W\rangle$ by measuring a suitable observable of a *single* probe particle that is prepared in an initial state ϱ_p and then interacts with the target. The evolution in the combined target-probe system from initial state ϱ to final state ϱ' is given by $\varrho' = T\varrho T^\dagger$, where the operator T depends on the Hamiltonian of the system under study. In terms of T the expectation value of a probe observable P in the final state reads $\langle P\rangle' = \mathrm{tr}\{P\varrho'\} = \langle T^\dagger P T\rangle$. Typically, P will be the projector onto a certain final probe state, *e.g.*, a photon's direction and polarization. In this case the expectation values of the (semi-definite) positive operator $T^\dagger P T$ cannot be negative, so that an entanglement witness seems nowhere in sight.

However, the probe particle can be in a superposition of states, for instance propagating through either of the two slits of a Young experiment. In other words, we allow that the evolution from initial to final state contains an interference between two exclusive alternatives labeled A and B. The phase difference $\phi_A - \phi_B = \phi$ between the interferometric path alternatives A and B should be under control, and the evolution is given in terms of $T = e^{i\phi_A}T_A + e^{i\phi_B}T_B$ with path-conditioned operators $T_{A,B}$.

Spelling out all contributions, the probe expectation value on the final state after interaction with the target becomes

$$\langle P\rangle' = \langle T_A^\dagger P T_A\rangle + \langle T_B^\dagger P T_B\rangle + \left[e^{i\phi}\langle T_B^\dagger P T_A\rangle + \mathrm{c.c.}\right]. \quad (1)$$

The first two addends are phase-independent and sum up to the usual background intensity I_0 of two-way interferometers. In particular, for a positive probe observable P these terms are also positive and by themselves useless for constructing an entanglement witness. But there are also the interference terms, which are responsible for fringes as function of the phase ϕ in the total detection intensity

$$\langle P\rangle' = I_0\left[1 + \mathcal{V}\cos(\phi - \alpha)\right], \quad (2)$$

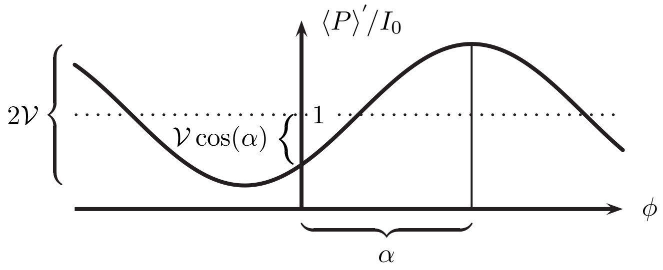

Fig. 1: Interference pattern of the probe expectation value. For a suitable choice of probe parameters, destructive interference at the origin $\phi = 0$ witnesses entanglement between the two target qubits.

drawn in fig. 1. The fringe visibility $\mathcal{V} = (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$ and the interaction-induced phase shift α are determined by the expectation value of the first cross-term in eq. (1), $2\langle T_B^\dagger P T_A\rangle = I_0\mathcal{V}e^{-i\alpha}$. The real part of this quantity, *i.e.*, the interference contribution at zero external phase shift $\phi = 0$, $I_0\mathcal{V}\cos\alpha$, is positive for constructive interference ($\cos\alpha > 0$) and negative for destructive interference ($\cos\alpha < 0$). It thus allows a sign change that we can exploit in order to define a witness. Under the condition that probe and target states are prepared independently, the initial state factorizes, $\varrho = \varrho_{12}\otimes\varrho_p$, and the interference contribution at the origin $\phi = 0$ can be rewritten as $\mathrm{tr}\{\varrho(T_B^\dagger P T_A + T_A^\dagger P T_B)\} = \mathrm{tr}_{12}\{\varrho_{12}M\}$, which is an expectation value of the target observable

$$M = \mathrm{tr}_p\big\{\varrho_p\big(T_B^\dagger P T_A + T_A^\dagger P T_B\big)\big\}. \quad (3)$$

This observable still depends on various quantities that can be chosen in order to realize an entanglement witness:

i) the interference path alternative A, B,

ii) the path-conditioned interaction operators $T_{A,B}$,

iii) the initial probe state ϱ_p, and

iv) the probe observable P.

The mapping (3) from the simple probe system to an observable of the composite target system constitutes the central idea of our letter, together with the explicit examples, given below, for probe parameters such that M is an entanglement witness. In essence, the initial state ϱ_{12} is entangled if the interference pattern (2) of the probe shows destructive interference at zero external phase shift. Both visibility and phase are easily extracted by fitting the experimental interference fringes. In this respect, the two-way interference pattern of a single probe permits to extract subtle quantum correlations between two given subsystems.

In the following, we will consider the witnesses

$$W_\pm = \mathbb{1} - 2|\Psi_\pm\rangle\langle\Psi_\pm|, \quad (4)$$

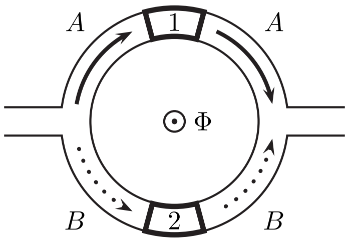

Fig. 2: Electronic Aharonov-Bohm interferometer with magnetic impurities embedded in each arm: A mesoscopic ring threaded by a magnetic flux Φ is attached to two supplying leads.

in terms of the Bell states $|\Psi_\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$, which are the singlet and triplet states of total spin zero. The maximal overlap between a separable state and a Bell state is 1/2. Consequently, any separable state yields a positive expectation value of $W_\pm$ so that a negative expectation value reliably indicates entanglement. In the remainder, we will show how these witnesses can be realized in mesoscopic solid-state devices by electron interference, or for atomic systems by photon interference. It turns out that the singlet entanglement can be distinguished relatively simply because of its distinct parity, but that it is much more difficult to distinguish the entangled triplet state from its separable neighbors within the triplet manifold.

Solid-state realization. – We shall first show that our concept allows to appreciate a proposal for probing entanglement between resonant quantum dots [9], within a rather lucid model description. Consider a single electron that probes the entanglement between two magnetic spin-1/2 impurities embedded in an Aharonov-Bohm (AB) ring [27,28], depicted in fig. 2: i) An electron propagating through arm A interacts only with the first impurity, whereas it interacts with the second impurity in arm B. The phase difference ϕ is controlled via the magnetic flux threading the ring, and the interference fringes are recorded by measuring the conductance of the ring.

We model the effective interaction between the electron spin and the two impurities $j = 1, 2$ by the isotropic spin-flip Hamiltonian

$$V_j = \hbar g \boldsymbol{\sigma} \cdot \boldsymbol{\tau}_j, \tag{5}$$

where $\boldsymbol{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ is the vector of Pauli matrices for the spin of the electron, and $\boldsymbol{\tau}_j$ are the impurity Pauli matrices. For this interaction type and geometry, the following choice of probe parameters ii)–iv) realizes the entanglement witness W_- of the singlet Bell state:

ii) In a symmetric geometry the probe interacts with both impurities an equal lapse of time t; the corresponding unitary time evolutions $T_A = \exp(-igt\boldsymbol{\sigma} \cdot \boldsymbol{\tau}_1)$ and $T_B = \exp(-igt\boldsymbol{\sigma} \cdot \boldsymbol{\tau}_2)$, respectively, read

$$T_{A,B} = \frac{\mathrm{e}^{\mathrm{i}gt}}{2} \left[\left(\mathrm{e}^{-\mathrm{i}2gt} + \cos 2gt\right) \mathbb{1} - \mathrm{i} \sin(2gt) \boldsymbol{\sigma} \cdot \boldsymbol{\tau}_{1,2} \right], \tag{6}$$

each with an interaction phase gt that we take as an experimentally tunable parameter. The partial probe trace of the cross-product $T_B^\dagger T_A$ is easily calculated to be

$$\mathrm{tr}_p\{T_B^\dagger T_A\} = \frac{1}{2} \left(|\mathrm{e}^{-\mathrm{i}2gt} + \cos 2gt|^2 \mathbb{1} + \sin^2(2gt) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right). \tag{7}$$

The choice $2gt = \pi/2$ immediately yields the singlet witness of (4) in the form $W_- = \frac{1}{2}(\mathbb{1} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$. Consequently, with iii) the probe spin in the unpolarized state $\varrho_p = \frac{1}{2}\mathbb{1}$ and probe detection without spin analysis, *i.e.*, measuring iv) the identity $P = \mathbb{1}$, the target operator (3) realizes the desired witness, $M = W_-$. Thus, we recover the prediction, derived in a more elaborate theoretical description [9], that AB current oscillations across a singlet-entangled double quantum dot show a characteristic minus sign.

The triplet witness of (4) can only be written as an *anisotropic* combination of Pauli matrices, $W_+ = \frac{1}{2}(\mathbb{1} - \tau_1^x\tau_2^x - \tau_1^y\tau_2^y + \tau_1^z\tau_2^z)$. Because both spatial symmetry of interaction and parity between the two impurities have to be broken, the authors of [9] proposed to apply an inhomogeneous magnetic field that provides the necessary (Berry) phase difference for the spin x- and y-components along one of the two arms. In our description, one could equivalently resort to tuning the coupling strengths separately for each spin component in the spin-flip interaction (5). The triplet witness can then be realized by choosing the same coupling phase $2gt = \pi/2$ as before for all spin components except the x- and y-components of only one of the impurities which should see a stronger coupling $2g't = 3\pi/2$ or the reversed sign $2g't = -\pi/2$.

But instead of requiring fine-tuned coupling strengths or supplementary control fields, we rather wish to realize the witness by measuring solely the probe particle. We have found that an effective witness for the triplet Bell state can be realized using an initially polarized electron state $\varrho_p = \frac{1}{2}(\mathbb{1} + \sigma^z)$ and measurement of the observable $P = \sigma^z$ such that $M = W_+ + \frac{1}{2}(\tau_1^z + \tau_2^z)$. Its expectation value in the Bell state $|\Psi_+\rangle$ is -1, whereas the expectation value in any separable state cannot be smaller than $-\frac{1}{4}$, which sets only a slightly lower threshold value for entanglement detection than zero. Furthermore, effective witnesses for the two other triplet Bell states $|\Phi_\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ are immediately obtained by a unitary rotation on ϱ_p and P, from σ^z to σ^x or σ^y, respectively.

Quantum optics realization. – In a second approach we examine interference of low-intensity laser light, *i.e.*, a single-probe photon scattered by two tightly trapped atoms on their resonant dipole transitions of total angular momentum $\frac{1}{2}$ in ground and excited state. In the absence of an external magnetic field, each degenerate ground state is an effective spin-$\frac{1}{2}$, such that the two atoms carry a qubit pair. This situation corresponds to the experiments by Eichmann *et al.* [29], who studied how the Young-fringe visibility disappeared when path knowledge was encoded in the atomic ground states. The internal states of the

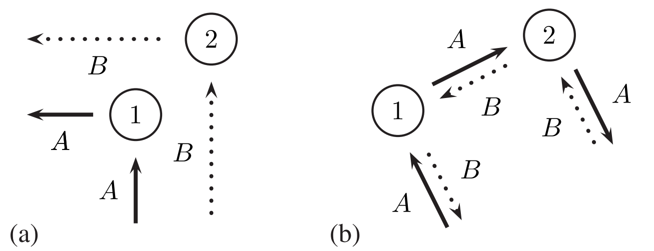

Fig. 3: Path alternatives for (a) Young interference and (b) coherent backscattering of a photon by two tightly trapped atoms.

atoms, however, were separable, and the possible influence of their entanglement was not studied.

The scattering of a photon with wave vector $\boldsymbol{k}$ and transverse polarization $\boldsymbol{\epsilon}\perp\boldsymbol{k}$ by a resonant atomic dipole transition is described, within the dipole coupling scheme, by the dyadic operator $\boldsymbol{d}\circ\boldsymbol{d}$. Acting on a pure spin-1/2 multiplet, the dipole vector operator $\boldsymbol{d}$ must be proportional to the only available vector operator, the spin operator itself. Thus we can take $\boldsymbol{d}=\boldsymbol{\tau}$, up to a frequency-dependent prefactor describing the interaction strength left implicit in the following. Photon scattering $(\boldsymbol{k}\boldsymbol{\epsilon})\mapsto(\boldsymbol{k}'\boldsymbol{\epsilon}')$ by a single atom is therefore described by $\bar{\boldsymbol{\epsilon}}'\cdot T_{A,B}\cdot\boldsymbol{\epsilon}=(\bar{\boldsymbol{\epsilon}}'\cdot\boldsymbol{\tau}_{1,2})(\boldsymbol{\tau}_{1,2}\cdot\boldsymbol{\epsilon})$ where $\bar{\boldsymbol{\epsilon}}'$ denotes the complex conjugate of the scattered polarization vector, and the scattering phase for an atom at the position $\boldsymbol{r}_j$ reads $\phi_j=(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}_j$, $j=1,2$.

i) We consider first the Young-type interference of fig. 3(a), in which the scattering atoms constitute the path alternatives. The phase difference is given by $\phi=(\boldsymbol{k}-\boldsymbol{k}')\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)$.

ii) Proceeding as in the solid-state realization above, we calculate the interference contribution for an unpolarized initial photon state $\varrho_p=\frac{1}{2}\mathbb{1}$ and no polarization analysis at detection $P=\mathbb{1}$,

$$\mathrm{tr}_p\{T_B^\dagger T_A\}=\sum_{\boldsymbol{\epsilon}'\perp\boldsymbol{k}'}\sum_{\boldsymbol{\epsilon}\perp\boldsymbol{k}}(\bar{\boldsymbol{\epsilon}}\cdot\boldsymbol{\tau}_2)(\boldsymbol{\tau}_2\cdot\boldsymbol{\epsilon}')(\bar{\boldsymbol{\epsilon}}'\cdot\boldsymbol{\tau}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\epsilon}). \quad (8)$$

The sums over polarization vectors yield tranverse projectors, $\sum_{\boldsymbol{\epsilon}\perp\boldsymbol{k}}\bar{\boldsymbol{\epsilon}}\circ\boldsymbol{\epsilon}=\mathbf{1}-\hat{\boldsymbol{k}}\circ\hat{\boldsymbol{k}}$, and the target observable (3) becomes

$$M=\left(1+(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}')^2\right)\mathbb{1}+\boldsymbol{\tau}_1\cdot\boldsymbol{M}\cdot\boldsymbol{\tau}_2, \quad (9)$$

with a dyadic $\boldsymbol{M}=\hat{\boldsymbol{k}}\circ\hat{\boldsymbol{k}}+\hat{\boldsymbol{k}}'\circ\hat{\boldsymbol{k}}'+(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}')\circ(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}')$. For detection perpendicular to incidence, $\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}'=0$, $\boldsymbol{M}$ becomes the sum of projectors onto the three orthogonal directions $\hat{\boldsymbol{k}}$, $\hat{\boldsymbol{k}}'$, and $\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}'$ such that $\boldsymbol{M}=\mathbf{1}$ is the 3×3 unit matrix. Consequently, the Young interference of iii) an unpolarized photon detected around right angles from the incident direction and iv) without polarization analysis, realizes the singlet witness up to an irrelevant multiplicative factor: $M=\mathbb{1}+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2=2W_-$.

In this example, detection perpendicular to incidence is necessary because all three spin components appearing in W_- can only be probed with tranverse photons if two directions of propagation are used. However, even for arbitrary directions of probe propagation and arbitrary polarization states at incidence and detection, the Young interference does not permit to realize the triplet witness.

In order to realize W_+ by a projective measurement, we have to go beyond the Young interference term, which is the leading single-scattering contribution in a general multiple-scattering expansion. To next order in the small parameter $1/k|\boldsymbol{r}_1-\boldsymbol{r}_2|$, we consider double scattering where the atoms exchange a single, intermediate virtual photon. There are now again two path alternatives, corresponding to the order of scattering events, see fig. 3(b). Characteristically, the phase difference $\phi=(\boldsymbol{k}+\boldsymbol{k}')\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)$ vanishes exactly for scattering in the backward direction $\boldsymbol{k}'=-\boldsymbol{k}$ such that the interference survives even an average over random positions. This type of interference explains both coherent backscattering (CBS) in optics and weak localization phenomena in mesoscopic electronic devices [30].

Recently, the role of which-path information for CBS by atoms with internal degeneracy [31] has been studied [32], but in the general case, no obvious signature of entanglement between scatterers in the interference pattern was found. In the meantime, however, we could show that CBS can indeed be sensitive to entanglement and allows to realize both witnesses $W_\pm$ with the following choice for the probe parameters:

i) For CBS interference, the path labels now describe the order in which the atoms are visited, *i.e.* $1\to2$ and $2\to1$, respectively, as shown in fig. 3(b).

ii) The transition operator for path A, $T_A=(\boldsymbol{\tau}_2\circ\boldsymbol{\tau}_2)\cdot(\mathbf{1}-\hat{\boldsymbol{n}}\circ\hat{\boldsymbol{n}})\cdot(\boldsymbol{\tau}_1\circ\boldsymbol{\tau}_1)$, is obtained by connecting the single-scattering transition operators by the far-field projector $(\mathbf{1}-\hat{\boldsymbol{n}}\circ\hat{\boldsymbol{n}})$ onto the plane transverse to the unit vector $\hat{\boldsymbol{n}}$ joining the two atoms. The operator for path B is obtained by exchanging the role of the two atoms, *i.e.*, by substituting $1\leftrightarrow2$.

iii) The probe photon has to impinge at right angles to the axis connecting the atoms ($\boldsymbol{k}\cdot\hat{\boldsymbol{n}}=0$) and again needs to be unpolarized.

iv) The detection around the backscattering direction $\boldsymbol{k}'=-\boldsymbol{k}$ can be made with a polarizing beam splitter whose two outcomes realize both witnesses simultaneously. The singlet witness W_- can be found in the channel of linear polarization along the unit vector $\hat{\boldsymbol{n}}$ joining the two atoms, and the triplet witness W_+ in the channel of linear polarization perpendicular to both $\hat{\boldsymbol{n}}$ and $\boldsymbol{k}$.

The CBS interference signal from two atoms can only be measured on top of the single-scattering background, which contributes with a geometry-dependent —and generally much larger— signal strength and unfortunately the same fringe spacing as double scattering. In the above configuration for the $W_\pm$ witnesses, the single-scattering signal is non-zero, with a (Young) interference visibility given by $\mathcal{V}=\frac{1}{2}(1+\langle\tau_1^z\tau_2^z\rangle)$ and no phase shift ($\alpha=0$). But whereas single scattering contributes in the completely mixed state with constructive interference ($\mathcal{V} = \frac{1}{2}$) completely masking the double-scattering contribution, its visibility vanishes in the Bell states $|\Psi_{\pm}\rangle (\mathcal{V} = 0)$. Thus, if fringes with destructive phase are observed, they will be the CBS signal for the given entanglement witnesses on top of the flat single-scattering background and therefore are the unambiguous signal of bipartite entanglement.

In summary, we have shown how the interference pattern of a single probe that interacts with a two-qubit quantum system can witness its bipartite entanglement, the signature being the change from constructive to destructive interference. We have found proof-of-principle models for realizations in standard solid-state and quantum-optics settings, which now await experimental realization and more quantitative calculations.

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We thank Ch. Miniatura and B.-G. Englert for pleasurable discussions. FM acknowledges financial support by Alexander von Humboldt foundation.

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