Single-probe interference can witness entanglement

Torsten Scholak   Cord Axel Müller

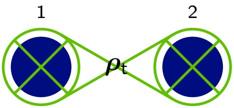
Quantum Transport of Light and Matter

UNIVERSITÄT BAYREUTH

YEP-07

Motivation

- task
  - detect **entanglement** in bipartite quantum system



- common limitations
  - multitude of observables necessary for entanglement characterization
  - entanglement witness: **single** observable, but also invokes several local measurements
  - only few measurements possible
- desire
  - characterize entanglement with **minimal** set of measurements

Proposed Solution

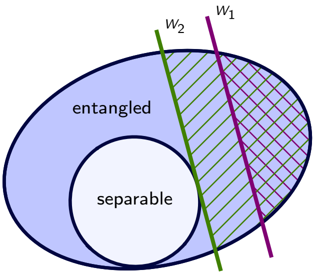
- let a probe particle gather the desired information

$\Rightarrow$ retrieve from final probe state

- no engineered interaction between 1 & 2

Theory of Entanglement & Entanglement Witnesses

- ρ_t **separable**$:\Leftrightarrow \rho_t = \sum_i p_i\, \rho_1^i \otimes \rho_2^i$
- otherwise: **entangled**



- an observable W is an **entanglement witness**$:\Leftrightarrow$

$$\langle W \rangle = \mathrm{tr}\{\rho_t W\} \begin{cases} \geq 0, & \text{for all separable states } \rho_t \\ < 0, & \text{for at least one entangled state} \end{cases}$$

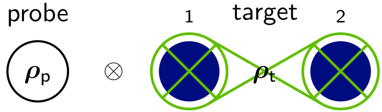
$\Rightarrow$ **negative** outcome: ρ_t entangled!

cf. P. Horodecki, Phys. Lett. A **232**, 33 (1997)

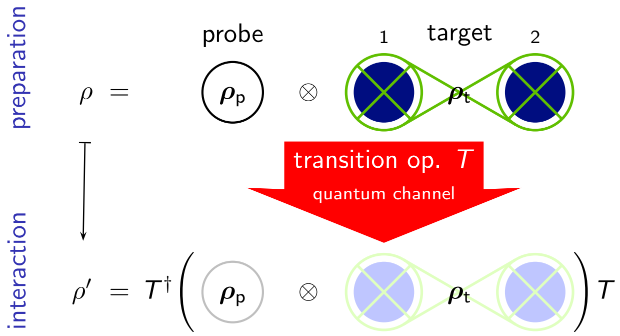
Simple Target-Probe Interaction

preparation

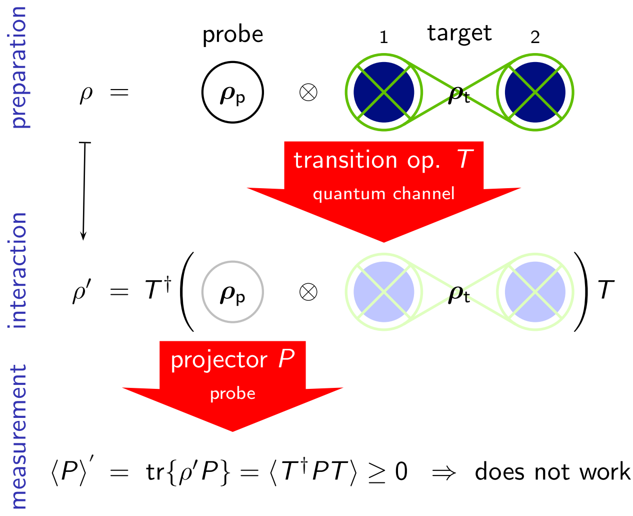
$\rho =$



Simple Target-Probe Interaction

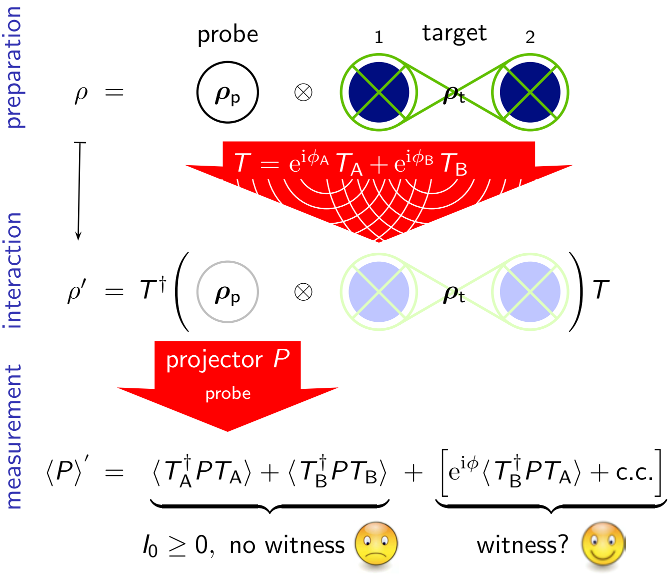


Simple Target-Probe Interaction



$$\langle P\rangle' = \operatorname{tr}\{\rho' P\} = \langle T^\dagger P T\rangle \geq 0 \;\Rightarrow\; \text{does not work as a witness}$$

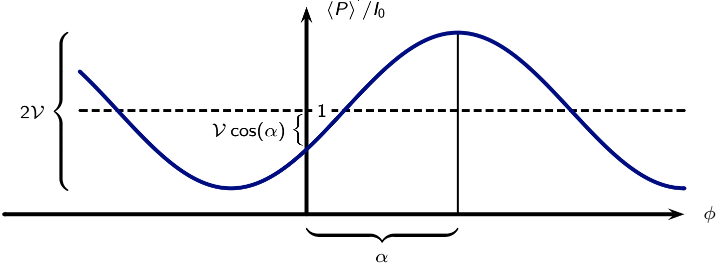
Refined Target-Probe Interaction with Interference



$\phi = \phi_\mathrm{A} - \phi_\mathrm{B}$

Interference Pattern

$$\langle P\rangle' = \langle T_A^\dagger P T_A + T_B^\dagger P T_B\rangle + \left[e^{i\phi}\langle T_B^\dagger P T_A\rangle + \text{c.c.}\right]$$
$$= I_0\left[1 + \mathcal{V}\cos(\phi - \alpha)\right]$$



- **fringe visibility:** $\mathcal{V} = \dfrac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \dfrac{2}{I_0}\, e^{-i\alpha}\langle T_A^\dagger P T_B\rangle$

Central Result: Connection of Pattern and **Witness**

⇒ **designated entanglement witness**:

reading off **interference** contribution at $\phi = 0$

$$\mathcal{V} I_0 \cos\alpha = \langle T_\mathrm{B}^\dagger P T_\mathrm{A} + T_\mathrm{A}^\dagger P T_\mathrm{B} \rangle$$

Central Result: Connection of Pattern and **Witness**

⇒ **designated entanglement witness**:

reading off
interference contribution
at $\phi = 0$

$$\underbrace{\mathcal{V} I_0 \cos\alpha}_{\text{reading off interference contribution at } \phi=0} = \mathrm{tr}_{\mathrm{p\&t}}\left\{(\rho_{\mathrm{p}} \otimes \rho_{\mathrm{t}})\left(T_{\mathrm{B}}^{\dagger} P T_{\mathrm{A}} + T_{\mathrm{A}}^{\dagger} P T_{\mathrm{B}}\right)\right\}$$

Central Result: Connection of Pattern and **Witness**

⇒ **designated entanglement witness**:

reading off **interference** contribution at $\phi = 0$

$$\mathcal{V} I_0 \cos\alpha \quad = \mathrm{tr}_\mathsf{t}\Big\{\rho_\mathsf{t} \ \ \mathrm{tr}_\mathsf{p}\Big[\rho_\mathsf{p}\Big(T_\mathsf{B}^\dagger P T_\mathsf{A} + T_\mathsf{A}^\dagger P T_\mathsf{B}\Big)\Big] \ \Big\}$$

Central Result: Connection of Pattern and **Witness**

$\Rightarrow$ **designated entanglement witness**:

reading off **interference** contribution at $\phi = 0$ $\Leftrightarrow$ measuring exp. value of **observable** on the target system

$$\mathcal{V} I_0 \cos\alpha = \mathrm{tr_t}\left\{\rho_\mathrm{t} \, \mathrm{tr_p}\left[\rho_\mathrm{p}\left(T_\mathrm{B}^\dagger P T_\mathrm{A} + T_\mathrm{A}^\dagger P T_\mathrm{B}\right)\right]\right\} = \mathrm{tr_t}\left\{\rho_\mathrm{t} M\right\}$$

Central Result: Connection of Pattern and **Witness**

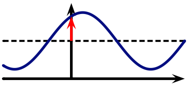
⇒ **designated entanglement witness**:

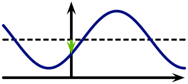
reading off **interference** contribution at $\phi = 0$	⇔	measuring exp. value of **observable** on the target system

$$\mathcal{V} I_0 \cos\alpha = \mathrm{tr}_\mathrm{t}\left\{\rho_\mathrm{t}\, \mathrm{tr}_\mathrm{p}\left[\rho_\mathrm{p}\left(T_\mathrm{B}^\dagger P T_\mathrm{A} + T_\mathrm{A}^\dagger P T_\mathrm{B}\right)\right]\right\} = \mathrm{tr}_\mathrm{t}\left\{\rho_\mathrm{t} M\right\}$$

⇒ **criterion:**

$\cos\alpha$

> 0 :  constructive interference 

< 0 :  destructive, ρ_t **entangled**! 

Obtaining the Witness

- controllable parameters
  - path alternatives A, B
  - path-conditioned interaction operators $T_{\mathrm{A,B}}$
  - initial probe state ρ_{p}, projector P

- bipartite **qubit** systems
- entanglement in the **singlet** Bell state

$$|\mathrm{S}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

- optimal witness

$$W_{\mathrm{S}} = \mathbb{1} - 2|\mathrm{S}\rangle\langle\mathrm{S}| = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

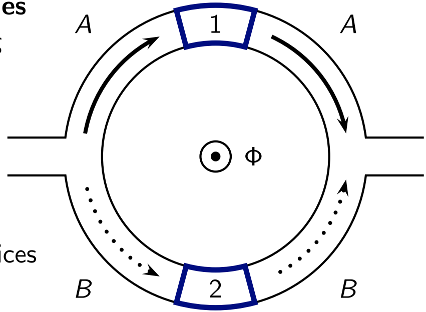
Solid-state Realization of W_S

- **probe** particle: single **electron**
- **target**: 2 magnetic spin-1/2 **impurities**
  - embedded in **Aharonov-Bohm ring**
- path alternatives: A = 1, B = 2
- isotropic **spin-flip** interaction

$$V_j = \hbar g \boldsymbol{\sigma} \cdot \boldsymbol{\tau}_j, \quad j = 1, 2$$

with $\left.\begin{matrix} \boldsymbol{\sigma} & \text{electron} \\ \boldsymbol{\tau}_j & \text{impurity} \end{matrix}\right\}$ spin Pauli matrices

- phase difference ϕ controlled via magnetic flux Φ



Specifying Parameters

- unitary time evolution $T_j = \exp(-\mathrm{i}gt\boldsymbol{\sigma}\cdot\boldsymbol{\tau}_j)$ with tunable phase gt
- $$\langle T_\mathsf{B}^\dagger T_\mathsf{A}\rangle = \frac{1}{2}\left(\left|\cos 2gt + \mathrm{e}^{-2\mathrm{i}gt}\right|^2 \mathbb{1} + \sin^2(2gt)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right)$$
- **unpolarized** electron state $\rho_\mathsf{p} = \frac{1}{2}\mathbb{1}$
- probe observable: **identity** $P = \mathbb{1}$

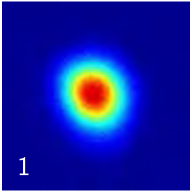
⇒ choice $2gt = \pi/2$ finally yields

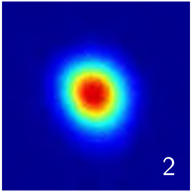
$$M = \frac{1}{2}\left(\mathbb{1} + \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right) = W_\mathsf{S}$$

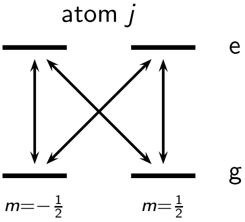
Quantum Optics Realization of W_S

- interference of low-intensity laser light, **single probe photon**
- elastic **scattering** by 2 tightly trapped atoms with $\frac{1}{2} \leftrightarrow \frac{1}{2}$ dipole transitions
- no magnetic field: each degenerate ground state is effective spin-$\frac{1}{2}$

$\Rightarrow$ 2 atoms carry **target** qubit pair







Lab Experiments by Eichmann et al. (1993)

- studied interference of **Young** type, i.e. single-scattering
- considered only **separable** internal states

PHYSICAL REVIEW LETTERS

VOLUME 70 | 19 APRIL 1993 | NUMBER 16

Young's Interference Experiment with Light Scattered from Two Atoms

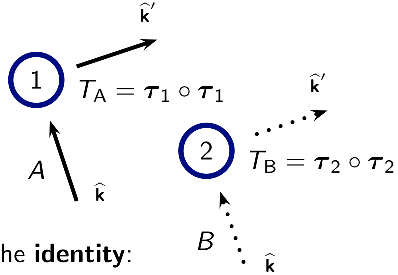
U. Eichmann,[(a)] J. C. Bergquist, J. J. Bollinger, J. M. Gilligan, W. M. Itano, and D. J. Wineland
National Institute of Standards and Technology, Boulder, Colorado 80303

M. G. Raizen
Department of Physics, University of Texas, Austin, Texas 78712
(Received 18 December 1992)

Specifying a Suitable Resonant Dipole Transition

- dipole coupling scheme
  - dyadic operator $\mathbf{d} \circ \mathbf{d}$
  - only one vector operator on spin-$\frac{1}{2}$ multiplet: $\boldsymbol{\tau}$
- $\Rightarrow$ $\mathbf{d} \propto \boldsymbol{\tau}$, omit prefactors

$\Rightarrow$



- **unpolarized** incident beam, measure the **identity**:

$$M = \mathrm{tr}_{\mathrm{p}} \left\{ \rho_{\mathrm{p}} \left(T_{\mathrm{B}}^{\dagger} P T_{\mathrm{A}} + T_{\mathrm{A}}^{\dagger} P T_{\mathrm{B}} \right) \right\}$$

$$= \frac{1}{2} \sum_{\widehat{\boldsymbol{\varepsilon}} \perp \widehat{\mathbf{k}}} \sum_{\widehat{\boldsymbol{\varepsilon}}' \perp \widehat{\mathbf{k}}'} \mathrm{tr}_{\mathrm{p}} \left\{ (\widehat{\boldsymbol{\varepsilon}} \circ \widehat{\boldsymbol{\varepsilon}}^{*})(\boldsymbol{\tau}_2 \circ \boldsymbol{\tau}_2)(\widehat{\boldsymbol{\varepsilon}}' \circ \widehat{\boldsymbol{\varepsilon}}'^{*})(\boldsymbol{\tau}_1 \circ \boldsymbol{\tau}_1) + \mathrm{h.c.} \right\}$$

- with $\widehat{\mathbf{k}} \perp \widehat{\mathbf{k}}'$ we obtain

$$M = \mathbb{1} + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = 2\, W_{\mathrm{S}}$$

Discussion and Outlook

- single-probe interference **can indeed witness** entanglement
- proof-of-concept models found in mesoscopics and quantum optics

- expectation values of witnesses also have some quantitative significance
- preparation of entanglement

- **Acknowledgments**: Florian Mintert, MPI-PKS Dresden

Thank You for Your Attention!